Day 3: Review of Motion Along a Line

My students study motion along a line early in the year, so this assignment is a review for them. I like to spend a day on motion along a line as a segue into motion along a curve. For an excellent introduction to motion along a line, see the Curriculum Module on motion by Dixie Ross at AP Central®. (http://apcentral.collegeboard.com/apc/public/repository/AP_CurricModCalculusMotion.pdf)

Day 3 Homework

The following problems are from old AP Exams and the sample multiple-choice problems in the Course Description, available at AP Central (http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2118.html).

Multiple-Choice Items:

1. 2003 AP Calculus AB Exam, Item 25 (no calculator):

A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only

(B) $t = 3$ only

(C) $t = \frac{7}{2}$ only

(D) $t = 3$ and $t = \frac{7}{2}$

(E) $t = 3$ and $t = 4$

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2. 1998 AP Calculus AB Exam, Item 24 (no calculator):
The maximum acceleration attained on the interval $0 \leq t \leq 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9

(B) 12

(C) 14

(D) 21

(E) 40

3. AP Calculus AB, sample multiple-choice Item 9 (no calculator):
The position of a particle moving along a line is given by $s(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \geq 0$.

For what values of t is the speed of the particle increasing?

(A) $3 < t < 4$ only

(B) $t > 4$ only

(C) $t > 5$ only

(D) $0 < t < 3$ and $t > 5$

(E) $3 < t < 4$ and $t > 5$

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4. 2003 AP Calculus AB Exam, Item 76 (calculator):
A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

(A) –2.016

(B) –0.677

(C) 1.633

(D) 1.814

(E) 2.97

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5. 2003 AP Calculus AB Exam, Item 91 (calculator):
A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln(1 + 2^t)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

(A) 0.462

(B) 1.609

(C) 2.555

(D) 2.886

(E) 3.346

6. AP Calculus AB, sample multiple-choice Item 19 (calculator):
Two particles start at the origin and move along the x-axis. For $0 \le t \le 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

(A) None

(B) One

(C) Two

(D) Three

(E) Four

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7. AP Calculus AB, sample multiple-choice Item 15 (calculator):

A particle travels along a straight line with a velocity of $v(t) = 3e^{\left(-t/2\right)} \sin(2t)$ meters per second. What is the total distance traveled by the particle during the time interval $0 \le t \le 2$ seconds?

(A) 0.835

(B) 1.850

(C) 2.055

(D) 2.261

(E) 7.025

Free-Response Questions:

8. 2004 AP Calculus AB Exam, FRQ 3 (calculator):
A particle moves along the y-axis so that its velocity at time $t \ge 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$.)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \ge 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

9. 2006 AP Calculus AB/BC Exams, Item 4 (no calculator):

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \dfrac{3}{\sqrt{t+1}}$ feet per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

Answers to Day 3 Homework

1. Since $x'(t) = 6t^2 - 42t + 72 = 6(t^2 - 7t + 12) = 6(t - 3)(t - 4) = 0$ when $t = 3$ and when $t = 4$, the answer is E.

2. Note that $a(t) = 3t^2 - 6t + 12$, so that $a'(t) = 6t - 6 = 0$ when $t = 1$. Computing the acceleration at the critical number and at the endpoints of the interval, we have $a(0) = 12$, $a(1) = 9$, and $a(3) = 21$. The maximum acceleration is 21, so the answer is D.

3. Note that $v(t) = 6t^2 - 48t + 90 = 6(t - 3)(t - 5)$ and $a(t) = 12t - 48 = 12(t - 4)$. The speed is increasing on $3 < t < 4$, where the velocity and the acceleration are both negative, and also for $t > 5$, where the velocity and the acceleration are both positive, so the answer is E.

4. Since $\dfrac{d}{dt}\left[3 + 4.1\cos(0.9t)\right]_{t-4} = 1.633$, the answer is C.

5. Since $v(2) = 2 + \int_1^2 \ln(1 + 2^t)\,dt = 3.346$, the answer is E.

6. First find $\frac{d}{dt}\left[\sin t\right] = \cos t$ and $\frac{d}{dt}\left[e^{-2t}\right] = -2e^{-2t}$. Then graph $y_1 = \cos x$ and $y_2 = -2e^{-2x}$ in function mode with an x-window of [0, 10] and a y- window of [–1 , 1]. The two graphs intersect at three points, so the answer is D.

7. Distance = $\int_0^2 \left|v(t)\,dt = \int_0^2 \left|3e^{\left(-t/2\right)} \sin\left(2t\right)\right|dt = 2.261\right.$, so the answer is D.

8. **(a)** $a(2) = v'(2) = -0.132$ or -0.133.

(b) $v(2) = -0.436$. Since $a(2) < 0$, and $v(2) < 0$, the speed is increasing.

(c) Note that $v(t) = 0$ when $\tan^{-1}(e^t) = 1$. The only critical number for y is $t = \ln(\tan 1) = 0.443$. Since $v(t) > 0$ for $0 \le t < \ln(\tan 1)$ and $v(t) < 0$ for $t > \ln(\tan 1)$, $y(t)$ has an absolute maximum at $t = 0.443$.

(d) $y\left(2\right) = -1 + \int_0^2 v\left(t\right)dt = -1.360$ or -1.361.

Since $v(2) < 0$ and $y(2) < 0$, the particle is moving away from the origin.

9. **(a)** Average acceleration of rocket A is

$$\frac{v\left(80\right) - v\left(0\right)}{80 - 0} = \frac{49 - 5}{80} = \frac{11}{20} \text{ ft / sec}^2.$$

(b) Since the velocity is positive, $\int_{10}^{70} v\left(t\right)dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds. A midpoint Riemann sum is

$$20\left[v\left(20\right) + v\left(40\right) + v\left(60\right)\right] = 20\left(22 + 35 + 44\right) = 2020 \text{ ft}.$$

(c) Let $v_B\left(t\right)$ be the velocity of rocket B at time t. Then $v_B\left(t\right) = \int \frac{3}{\sqrt{t+1}}dt = 6\sqrt{t+1} + C$. Since $2 = v_B\left(0\right) = 6 + C$, then C = – 4 and $v_B\left(t\right) = 6\sqrt{t+1} - 4$. Hence, $v_B\left(80\right) = 50 > 49 = v\left(80\right)$ and Rocket B is traveling faster at time $t = 80$ seconds.